Math 10B with Professor Stankova
Quiz 11; Tuesday, 4/10/2018
Section #211; Time: 11 AM
GSI name: Roy Zhao **Name:** ____________________

Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. True **FALSE** It is possible for a BVP to have exactly 2 solutions.

> **Solution:** A BVP can have $0, 1$, or infinitely many solutions and no other option.

2. **TRUE** False If y_1, y_2 are two solutions to a linear homogeneous differential equation, then $y_1 + y_2$ is.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (5 points) Find the general solution to $y'' + 2y' + 2y = 0$.

> **Solution:** This is a second order linear homogeneous differential equation with constant coefficients. So we can solve it by solving the characteristic equation which is $r^2 + 2r + 2 = 0$ and that gives us $r = -1 \pm i$. Therefore, the general solution is
> $$y(t) = c_1 e^{-t} \cos(t) + c_2 e^{-t} \sin(t).$$

(b) (4 points) Give an IVP involving a second order differential equation such that $y(t) = e^{2t} + e^t$ is a solution.

> **Solution:** Since we have e^{2t} and e^t, this tells us that the roots are $r = 1, 2$ and hence the characteristic equation is $(r-1)(r-2) = r^2 - 3r + 2 = 0$. So the differential equation is $y'' - 3y' + 2y = 0$. The initial conditions are $y(0) = e^0 + e^0 = 2$ and $y'(0) = 2e^0 + e^0 = 3$.

(c) (1 point) Prove that $\tan(\theta) = \frac{1}{i} \cdot \frac{e^{i\theta} - e^{-i\theta}}{e^{i\theta} + e^{-i\theta}}$.

> **Solution:** We use Euler's formula which says that $e^{i\theta} = \cos(\theta) + i\sin(\theta)$. Then, we have that
> $$\frac{1}{i} \cdot \frac{e^{i\theta} - e^{-i\theta}}{e^{i\theta} + e^{-i\theta}} = \frac{1}{i} \frac{\cos(\theta) + i\sin\theta - (\cos(-\theta) + i\sin(-\theta))}{\cos(\theta) + i\sin\theta + (\cos(-\theta) + i\sin(-\theta))}$$

Now we use that cos is even and sin is odd to get that $\cos(-\theta) = \cos(\theta)$ and $\sin(-\theta) = -\sin(\theta)$ so the above is

$$= \frac{1}{i}\frac{2i\sin(\theta)}{2\cos(\theta)} = \frac{\sin\theta}{\cos\theta} = \tan\theta.$$